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A REGENERATOR—PREDICTION OF NUSSELT NUMBERS

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Abstract—The dynamics of a parallel plate regenerative heat exchanger are studied using a model in which resistance to heat transfer is due to diffusional resistance in the fluid in the direction transverse to flow. The resultant Nusselt number is a function of time as well as longitudinal distance from the entrance of the exchanger and is not a constant, as is assumed in the traditional regerator problem. However, for reasonably large dimensionless longitudinal distances from the entrance, the temperature profiles agree reasonably well with those obtained from the traditional Anzelius–Schumann solution despite the fact that the local Nusselt numbers become very large for large times.

NOMENCLATURE

$B_1(u)$, function defined by equation (A.14);
$B_2(u)$, function defined by equation (A.15);
C, solute concentration in fluid phase [g/cm³];
C_{in}, fluid phase inlet solute concentration [g/cm³];
$C_{i.\,wall}$, initial fluid phase concentration adjacent to the plate and in equilibrium with solute in adsorbed phase on the plate [g solute/cm³];
C_{ps}, heat capacity of the plate [cal/g °K];
D, molecular diffusion coefficient of solute in fluid phase [cm²/s];
h, heat-transfer coefficient [cal/s cm² °K], or mass-transfer coefficient [cm/s];
i, $\sqrt{-1}$;
k, thermal conductivity of fluid phase [cal/s cm °K];
K, Henry's law type equilibrium coefficient,

$$\frac{\text{g adsorbed solute/cm}^3\text{ adsorbed phase}}{\text{g solute/cm}^3\text{ fluid phase}};$$

L, characteristic length, half distance between the plates [cm];
m_s, mass of plate per unit area of plate surface [g/cm²]; or mass of adsorped phase per unit area of plate surface [g/cm²];
Nu, Nusselt number, defined by equation (15), dimensionless;
s, Laplace transform variable;
t, dimensionless time, $kt_A/m_sC_{ps}L$ for heat-transfer case, Dt_A/m_sKL for mass-transfer case;
t_A, actual time [sec];
T, temperature [°K];
T_b, bulk temperature of the fluid phase, defined by equation (14) [°K];
T_{in}, fluid phase inlet temperature [°K];
$T_{i.\,wall}$, initial plate temperature [°K];
u, defined by equation (A.12) and used as variable of integration;
U, dimensionless temperature or dimensionless concentration,

$$\frac{T - T_{in}}{T_{i.\,wall} - T_{in}} \text{ or } \frac{C - C_{in}}{C_{i.\,wall} - C_{in}};$$

U_b, dimensionless bulk temperature or concentration, $\int_0^1 U(x, t, y)\,dy$
U_{solid}, dimensionless temperature or dimensionless equilibrium concentration of the solid phase;

v, constant plug flow velocity of fluid phase;
x, dimensionless longitudinal coordinate, $\alpha x_A/vL^2$ for heat-transfer case, Dx_A/vL^2 for mass-transfer case also, real part of complex number;
x_A, actual longitudinal coordinate [cm];
y, dimensionless transverse coordinate, y_A/L also, imaginary part of complex number Z;
y_A, actual transverse coordinate [cm];
z, complex number.

Greek symbols

α, thermal diffusivity of fluid phase [cm^2/s];
η, same as y;
ξ, same as x;
$\varphi_R(u, \eta)$, function defined by equation (A.17);
$\varphi_I(u, \eta)$, function defined by equation (A.18);
χ, transfer coefficient in equation (16), as used by Ackermann [1].

INTRODUCTION

In most mathematical models formulated to describe the dynamics of heat regenerators or mass exchangers the resistance to transfer between solid and fluid phases is accounted for by means of a constant transfer coefficient [2, 9, 13]. An extensive discussion of these solutions, with references, is given by Jakob [8]. The analogous mass transfer cases are discussed by Hougen and Watson [6]. Lightfoot [10] discusses some of these models as pertains to ion exchange, chromatography and drying.

The purpose of this paper is to examine the assumption that the Nusselt number is a constant. The model considered attributes heat transfer resistance to conduction in the fluid phase transverse to the direction of flow. Thermal conductivities in the solid phase are assumed infinite in the transverse direction and the conduction of heat in the direction parallel to flow is neglected compared with the transfer by convection. These are the usual assumptions made in connection with regenerator studies [8]. To facilitate solution of the problem, a constant and uniform velocity profile in the fluid phase is assumed, i.e. plug flow. This assumption follows Sparrow and Spalding [16] and Siegel [14] in that it is felt that the essential nature of the results will not be changed markedly. Also, in the entrance region plug flow can be assumed because a parabolic velocity profile has not yet been established.

MATHEMATICAL MODEL

Consider a parallel plate channel through which a fluid of constant physical properties is flowing in plug flow. The energy equation can be written as

$$v\frac{\partial T}{\partial x_A} - \alpha\frac{\partial^2 T}{\partial y_A^2} = 0. \tag{1}$$

Longitudinal diffusional effects can be shown to be negligible for reasonably high fluid flows, i.e. Péclet number greater than 10 [7]. Hence, they are neglected in equation (1). Since velocity is reasonably high, the accumulation term $\partial T/\partial t_A$ may also be omitted from equation (1). However, for the case of plug flow the transformation to a characteristic type coordinate [4, 8, 14] will eliminate the accumulation term. At the plate wall we assume that all the heat leaving the fluid goes to increase the enthalpy of the plate. This results in

$$k\frac{\partial T}{\partial y_A}\bigg|_{y_A=0} = m_s C_{ps}\frac{\partial T}{\partial t_A}\bigg|_{y_A=0} \tag{2}$$

No transfer across the centreline yields

$$\frac{\partial T}{\partial y_A}\bigg|_{y_A=L} = 0. \tag{3}$$

To transform the equations into dimensionless form we define the following dimensionless variables

$$t = \frac{kt_A}{m_s C_{ps} L} \tag{4}$$

$$y = \frac{y_A}{L} \tag{5}$$

$$x = \frac{\alpha}{vL^2} \cdot x_A \quad (6)$$

$$U = \frac{T - T_{in}}{T_{i.\,wall} - T_{in}} \quad (7)$$

Equations (1) and (2) transform into

$$\frac{\partial U}{\partial x} - \frac{\partial^2 U}{\partial y^2} = 0 \quad (8)$$

$$\left.\frac{\partial U}{\partial y}\right|_{y=0} = \left.\frac{\partial U}{\partial t}\right|_{y=0} \quad (9)$$

Equation (3) becomes

$$\left.\frac{\partial U}{\partial y}\right|_{y=1} = 0. \quad (10)$$

For boundary conditions we consider the case of constant initial wall temperature and constant inlet fluid temperature. These boundary conditions become

$$U(0, t, y) = 0 \quad (11)$$

$$U(x, 0, 0) = 1. \quad (12)$$

The heat-transfer coefficient h is defined by

$$-k\left.\frac{\partial T}{\partial y_A}\right|_{y_A=0} = h(T|_{y_A=0} - T_b) \quad (13)$$

where T_b is bulk average temperature defined as

$$T_b = \frac{1}{L}\int_0^L T(x_A, t_A, y_A)\,dy_A. \quad (14)$$

In dimensionless form we obtain the definition of the Nusselt number as

$$Nu = \frac{hL}{k} = \frac{\partial U/\partial y|_{y=0}}{U_b - U|_{y=0}}. \quad (15)$$

The dimensionless parameters for the analogous mass transfer problem are defined in the Nomenclature.

From equation (15) it is apparent that the Nusselt number is a function of time as well as longitudinal distance x.

MATHEMATICAL SOLUTION

A solution to equations (8) through (12) does not appear to have been reported previously in the literature. For spherical geometry Rosen [12] presents a solution to the same mathematical problem equivalent to the wall temperature $U(x, t, 0)$. In Rosen's physical model, however, diffusion within the solid spherical particles is considered rather than diffusion within the fluid phase. Rosen does not present the complete solution $U(x, t, y)$, which is needed here to determine the fluid bulk temperatures, $U_b(t, x)$. For the case where there exists a resistance between fluid and solid phase, equation (9) is replaced by

$$-\left.\frac{\partial U}{\partial y}\right|_{y=0} = \chi(U_{solid} - U|_{y=0}) \quad (16)$$

$$-\frac{\partial U_{solid}}{\partial t} = \chi(U_{solid} - U|_{y=0}). \quad (17)$$

Ackermann [1] solved the system of equations (8) through (10), (16), and (17) for what is equivalent to constant initial wall temperature and arbitrary inlet temperature $U(0, t, y)$. He obtained the solution in the form of an integral equation which was then solved by successive graphical integration.

The solutions presented in this paper to equations (8) through (12) were obtained by Laplace transforms and contour integration in the complex domain to invert the Laplace transform. This is similar to the procedure followed by Rosen [12]. For details of the present solution see Appendix A. The resultant solution for $U(x, t, y)$ is

$$U(x, t, y) = \frac{1}{2} + \frac{2}{\pi}\int_0^\infty \frac{1}{u}\exp[-tB_1(u)] \,.\, \{\sin(2xu^2 - tB_2(u)) \,.\, \varphi_R(u, y) + \cos[2xu^2 - tB_2(u)] \,.\, \varphi_I(u, y)\}\,du \quad (18)$$

where

$$B_1(u) = \frac{u(\sinh 2u - \sin 2u)}{\cosh 2u + \cos 2u} \tag{19}$$

$$B_2(u) = \frac{u(\sinh 2u + \sin 2u)}{\cosh 2u + \cos 2u} \tag{20}$$

$$\varphi_R(u, y) = \frac{\cosh yu \cos (2 - y)u + \cos yu \cosh (2 - y)u}{\cosh 2u + \cos 2u} \tag{21}$$

$$\varphi_I(u, y) = -\frac{\sinh yu \sin (2 - y)u + \sin yu \sinh (2 - y)u}{\cosh 2u + \cos 2u} \tag{22}$$

A computer program was written to evaluate equation (18) using Gaussian quadrature. It can be shown that for $x < 0{\cdot}044$ or for $t > 9x$ a boundary layer or penetration type solution is valid which is obtained by letting the channel width to be infinite:

$$U(x, t, y) = \operatorname{erfc} \frac{t + y}{2\sqrt{x}}. \tag{23}$$

Equation (23) satisfies the partical differential equation and all the boundary conditions except (10). However, for the region $x < 0{\cdot}044$, the dimensionless gradient at $y = 1$ is less than 0·01; and for $t > 9x$ the ratio of the gradient at $y = 1$ to the gradient at $y = 0$ is less than 0·01. Under these conditions it is felt that equation (23) satisfied (10) sufficiently well so that for all practical purposes equation (23) can be considered a solution. The complimentary error function solution was used to check some of the numerical results obtained from equation (18) at an x coordinate of 0·0444. The maximum absolute difference between the two different forms of solution was less than 0·0004.

Using equations (18) and (15) we obtain as an expression for the Nusselt number

$$Nu = \frac{-\int_0^\infty \frac{1}{u} \exp[-tB_1(u)] . [\sin A(x, t, u) . B_1(u) + \cos A(x, t, u) . B_2(u)] \, du}{\int_0^\infty \frac{1}{u} \exp[-tB_1(u)] . \left\{\sin A(x, t, u) . \left[\frac{B_2(u)}{2u^2} - 1\right] - \cos A(x, t, u) . \frac{B_1(u)}{2u^2}\right\} du} \tag{24}$$

where

$$A(x, t, u) = 2xu^2 - tB_2(u).$$

For low values of t, equation (24) is difficult to evaluate numerically. However, when $t = 0$ the boundary conditions to equation (8) become

$$\begin{aligned} U &= 0 \quad \text{for} \quad x = 0 \\ U &= 1 \quad \text{for} \quad y = 0. \end{aligned} \tag{25}$$

The solution of which is given in Carslaw and Jaeger [3]. Thus, this latter solution can be used to obtain the Nusselt number as a function of x for zero t.

BEHAVIOUR OF NUSSELT NUMBER AND TEMPERATURE

Figure 2 shows a logarithmic plot of Nusselt number against longitudinal coordinate x with

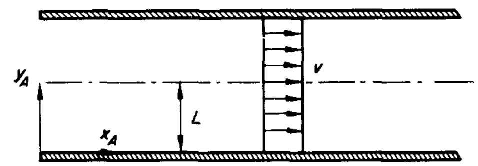

FIG. 1. Coordinate system for parallel plate channel.

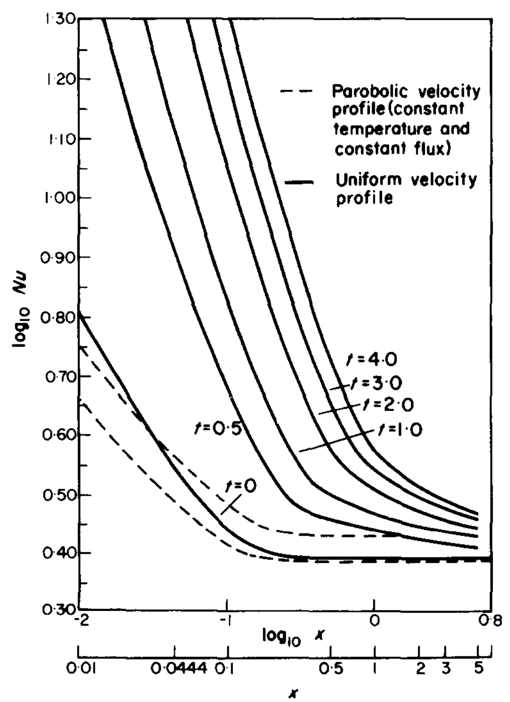

FIG. 2. Nusselt numbers for a regenerator.

time as a parameter. Using the asymptotic solution, equation (23), it can be shown that for a fixed x the Nusselt number increases without bound as time goes to infinity. As longitudinal distance x tends towards infinity for a fixed time, the Nusselt number tends toward the constant value of 2·467, which corresponds to the value obtained at zero time for reasonably large x. In other words, the curves in Fig. 2 converge towards this value. The dashed lines represent steady state Nusselt numbers calculated by Solbrig and Gidaspow [15] for the two cases of (1) constant wall temperature, and (2) constant flux at the wall. Their calculations were made assuming a parabolic velocity profile. The curve for zero t and the lower dashed line represent the same physical situation except that the assumed velocity profiles are respectively plug flow and parabolic. Their close agreement for a dimensionless x coordinate greater than about 0·25 supports the hypothesis that Nusselt number results calculated assuming plug flow are representative of the results that would be obtained if a more realistic velocity profile were assumed. Clearly, boundary conditions affect the Nusselt number more than does the velocity profile.

Since the results obtained here indicate that Nusselt numbers for regenerators are not constant at a given longitudinal coordinate but vary with time, the question naturally arises as to how serious is the error that is obtained in assuming a constant Nusselt number to calculate temperature profiles using the classical Anzelius–Schumann solution [9]. Figure 3 compares temperature profiles obtained from equation (18) with profiles obtained from the Anzelius–Schumann curves [9] using a constant Nusselt number of 2·467. This value for Nusselt number is a logical *a priori* value since it is the theoretical steady value for laminar plug flow with a constant wall temperature. As can be seen from Fig. 3 there is some discrepancy between the profiles. Figure 4 is the same type plot but based on a Nusselt number of 3. For this case, closer agreement is obtained between the two different

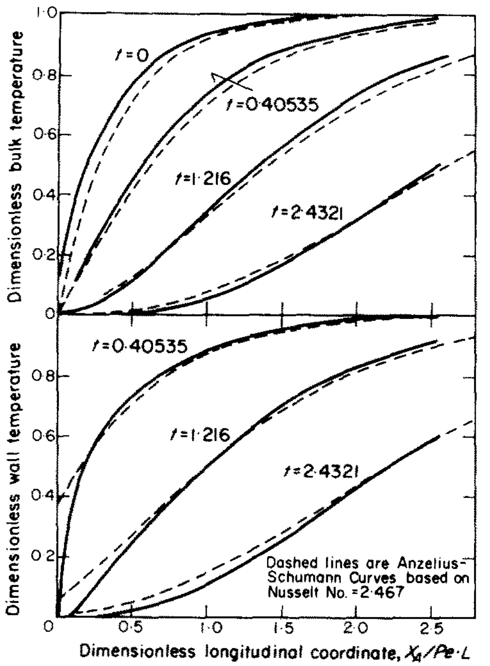

FIG. 3. Comparison with Schumann profiles based on developed Nusselt number.

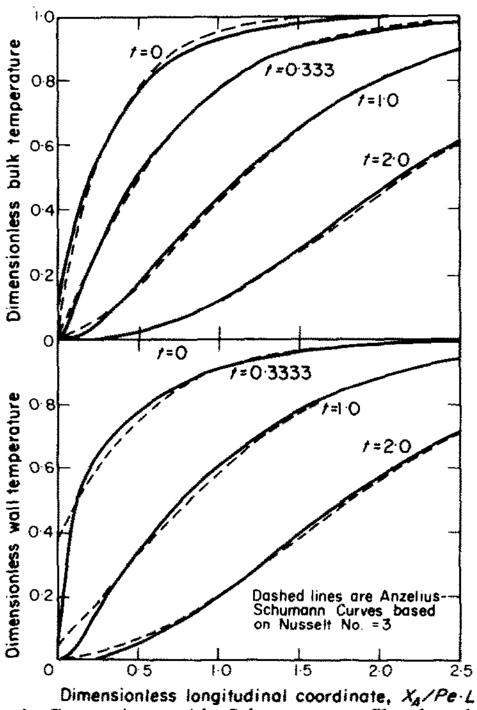

FIG. 4. Comparison with Schumann profiles based on Nusselt number of 3.

solutions indicating that for most practical purposes the assumption of a constant Nusselt number is not a bad one· at least in those instances where the dimensionless x values under consideration are relatively large. Such would usually be the case when the fluid phase is a gas. Major disagreement between the Anzelius–Schumann solution and the present solution occurs in the values for wall temperature at low

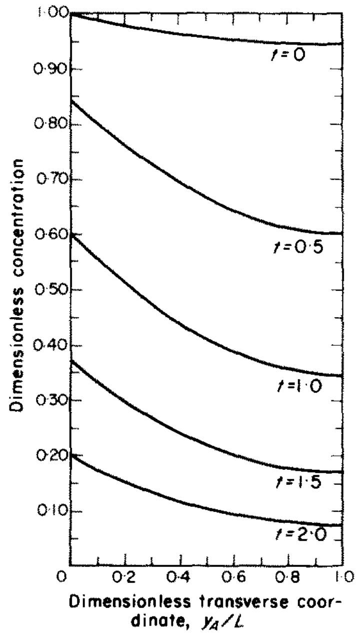

FIG. 5. Transverse temperature or concentration profiles at dimensionless longitudinal coordinate $x = 1$.

values of x. For small values of x in our model the Nusselt number becomes very large, as is true in every Graetz problem; therefore, the wall temperature at $x = 0$ becomes equal to the inlet temperature. In Schumann's analysis [13], however, the resistance to heat transfer remains constant throughout the length of the channel. This gives rise to a wall temperature at zero x different from the inlet temperature, which of course contradicts the usual observation, except for those cases when back conduction is important. The present model is believed to be more realistic near the entrance region than the Anzelius–Schumann model.

If desired, transverse temperature profiles within the fluid phase may be calculated using equation (18). Figure 5 shows how these profiles vary with time at a fixed longitudinal coordinate of x of one. It will be noticed that a transverse temperature distribution exists at zero t. Mathematically we are not free to specify the temperature throughout when the variable t equals zero because the system of equations would then be over determined. At a given longitudinal station, say $x = \xi$, time t is counted from the moment the disturbance which was introduced at x equals zero reaches the point $x = \xi$ that is, ξ/v time units after the fluid was introduced at x equals zero. During the period that this front traveled from $x = 0$ to $x = \xi$ it was receiving heat from the wall, which was at the initial dimensionless temperature of one. Thus we obtain an initial transverse temperature distribution which will be different at each x coordinate. Siegel [14] gives an excellent discussion of this phenomenon.

CONCLUSIONS

(1) A mathematical model for a regenerator has been proposed in which resistance to heat transfer is assumed to be due to the finite thermal conductivity of the fluid phase. Exact solutions for the temperature profile and the Nusselt number are presented as integrals which can be easily evaluated by computer using Gaussian quadrature.

(2) It was found that the Nusselt number does not reach a steady state value with time but continues to increase indefinitely. Thus, Nusselt numbers range from those obtained for the case of constant wall temperature to infinity. Comparison of the temperature profiles with those calculated from the Anzelius–Schumann curves [9] based on a constant Nusselt number of 3 indicates, however, that the simpler Anzelius–Schumann solution may be used without serious error except in the entrance, where the wall temperature as predicted by the Anzelius–Schumann curve is in error.

ACKNOWLEDGEMENT

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REFERENCES

1. G. Ackermann, Die Theorie der Wärmeaustauscher mit Warmespeicherung, *Z. Angew. Math. Mech.* **11**, 192–205 (1931).
2. A. Anzelius, Über Erwarmung vermittals durchströmender Medien, *Z. Angew. Math. Mech.* **6**, 291–294 (1926).
3. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd Edn, p. 96. Oxford University Press, London (1959).
4. C. A. Chase, Jr., Unsteady coupled heat and mass transfer with bulk flow, Ph.D. thesis, Illinois Institute of Technology, Chicago (1968).
5. R. V. Churchill, *Operational Mathematics*, 2nd Edn, p. 177. McGraw-Hill, New York (1958).
6. O. A. Hougen and K. M. Watson, *Chemical Process Principles*, Part III, pp. 1083–1085. John Wiley, New York (1953).
7. Chia-Jung Hsu, An exact mathematical solution for entrance-region laminar heat transfer with axial conduction, *Appl. Sci. Res.* **17**, 359–376 (1967).
8. M. Jakob, *Heat Transfer*, Vol. 2, pp. 275–314. John Wiley, New York (1957).
9. F. W. Larsen, Rapid calculation of temperature in a regenerative heat exchanger having arbitrary initial solid and entering fluid temperatures, *Int. J. Heat Mass Transfer* **10**, 149–168 (1967).
10. E. N. Lightfoot, R. J. Sanchez-Palma and D. O. Edwards, Chromatography and allied fixed-bed separations processes, *New Chemical Engineering Separation Techniques*, pp. 99–181. Interscience, New York (1962).
11. L. L. Pennisi, *Elements of Complex Variables*, p. 107. Holt, Rinehart & Winston, New York (1963).
12. J. B. Rosen, Kinetics of a fixed bed system for solid diffusion into spherical particles, *J. Chem. Phys.* **20**, 387–394 (1952).
13. T. E. W. Schumann, Heat transfer: A liquid flowing through a porous prism, *J. Franklin Inst.* **208**, 405 (1929).
14. R. Siegel, Transient heat transfer for laminar slug flow in ducts, *Trans. Am. Soc. Mech. Engrs—J. Appl. Mech.* (E) **26**, 140–142 (1959).
15. C. W. Solbrig and D. Gidaspow, Convective diffusion in a parallel plate duct with one catalytic wall—laminar flow—first order reaction, *Can. J. Chem. Engng* **45**, 35–39 (1967).
16. E. M. Sparrow and E. C. Spalding, Coupled laminar heat transfer and sublimation mass transfer in a duct, *Trans. Am. Soc. Mech. Engrs, J. Heat Transfer* (C) **90**, 115–124 (1968).

APPENDIX A

We consider U to be a three place function of x, t, and y, i.e. $U(x, t, y)$. We want to solve the system of equations

$$\frac{\partial U}{\partial x} - \frac{\partial^2 U}{\partial y^2} = 0 \quad \text{(A.1)}$$

$$\left.\frac{\partial U}{\partial y}\right|_{y=0} = \left.\frac{\partial U}{\partial t}\right|_{y=0} \quad \text{(A.2)}$$

$$\left.\frac{\partial U}{\partial y}\right|_{y=1} = 0 \quad \text{(A.3)}$$

$$U(x, 0, 0) = 1 \quad \text{(A.4)}$$

$$U(0, t, y) = 0. \quad \text{(A.5)}$$

Let $\bar{U}(s, t, y)$ denote the Laplace transform with respect to x of $U(x, t, y)$. In the Laplace domain we obtain as the solution of (A.1) through (A.5)

$$\bar{U}(s, t, y) = \frac{1}{s}(\cosh\sqrt{s}y - \tanh\sqrt{s}\sinh\sqrt{s}y) \times \exp(-t\sqrt{s}\tanh\sqrt{s}). \quad \text{(A.6)}$$

For convenience in what follows, denote x by ξ and y by η. Henceforth we will refer to $U(x, t, y)$ as $U(\xi, t, \eta)$. To determine $U(\xi, t, \eta)$ we will invert (A.6) by means of the inversion theorem. Let $f(s)$ denote $\bar{U}(s, t, \eta)$, where the dependence of t and η has been suppressed. Similarly let $F(\xi)$ denote $U(\xi, t, \eta)$. Applying the inversion theorem gives

$$F(\xi) = \frac{1}{2\pi i}\lim_{\beta\to\infty}\int_{\gamma-i\beta}^{\gamma+i\beta} e^{z\xi} f(z)\,dz \quad \text{(A.7)}$$

where γ is chosen such that $f(z)$ is analytic for $Re(z) > \gamma$. We see that $f(z)$ is analytic for all z such that $Re(z) > 0$. At $z = 0$ we have an essential singularity. Using the path of integration shown in Fig. A.1, $F(t)$ becomes (where $z = x + iy$, x and y are here the real and imaginary parts of the complex number z)

$$F(\xi) = \lim_{\epsilon\to 0}\left\{\frac{1}{2\pi i}\int_{-\infty}^{-\epsilon} e^{iy\xi} f(iy)\,i\,dy + \frac{1}{2\pi i}\int_{\epsilon}^{\infty} e^{iy\xi} f(iy)\,i\,dy + \frac{1}{2\pi i}\int_{\Gamma} e^{z\xi} f(z)\,dz\right\} \quad \text{(A.8)}$$

where Γ is the semicircle of radius ϵ

First, we will evaluate the third term on the right in equation (A.8).

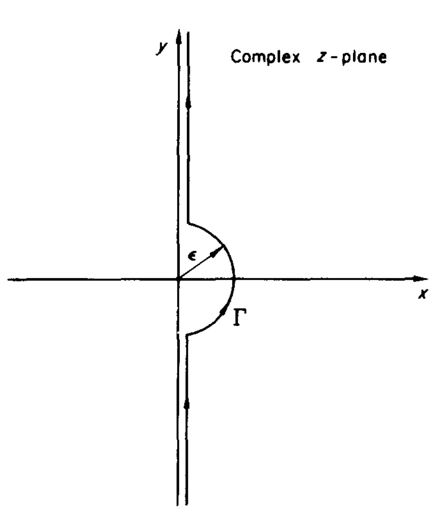

FIG. A.1. Integration contour in the complex plane.

$$\frac{1}{2\pi i}\int_{\Gamma} e^{z\xi} f(z)\,dz = \frac{1}{2\pi i}\int_{-\pi/2}^{\pi/2} \exp(\epsilon\xi\, e^{i\theta})\, f(\epsilon\, e^{i\theta})\, \epsilon i\, e^{i\theta}\,d\theta = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \frac{1}{\epsilon\, e^{i\theta}}\varphi(\epsilon\, e^{i\theta}, \eta)\exp\{\xi\epsilon\, e^{i\theta} - t(\sqrt{\epsilon})\, e^{i\theta/2}\tanh[(\sqrt{\epsilon})\, e^{i\theta/2}]\}\,\epsilon\, e^{i\theta}\,d\theta$$

$$\to \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2}\varphi(0, \eta)\,d\theta = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} d\theta = \frac{1}{2} \quad \text{as} \quad \epsilon \to 0. \quad \text{(A.9)}$$

Hence we have

$$\lim_{\epsilon\to 0}\frac{1}{2\pi i}\int_{\Gamma} e^{z\xi} f(z)\,dz = \tfrac{1}{2}. \quad \text{(A.10)}$$

Noting that

$$\frac{1}{2\pi}\int_{-\infty}^{-\epsilon} e^{iy\xi} f(iy)\,dy = \frac{1}{2\pi}\int_{\epsilon}^{\infty} e^{-iy\xi} f(-iy)\,dy$$

we obtain as shown in Churchill [5]

$$F(\xi) = \frac{1}{2} + \lim_{\epsilon\to 0}\frac{1}{2\pi}\int_{\epsilon}^{\infty}[e^{iy\xi} f(iy) + e^{-iy\xi}$$

$$f(-iy)]\,dy = \frac{1}{2} + \lim_{\epsilon\to 0}\frac{1}{\pi}\int_{\epsilon}^{\infty}\mathrm{Re}[e^{iy\xi} f(iy)]\,dy \qquad (A.11)$$

where

$$e^{iy\xi} f(iy) = \varphi(iy,\eta)\frac{1}{iy}\exp[iy\xi - t\sqrt{(iy)}\tanh\sqrt{(iy)}]$$

now

$$\sqrt{(iy)} = \sqrt{(y/2)}(1+i) = u(1+i)$$

where

$$u = \sqrt{(y/2)}. \qquad (A.12)$$

Employing the fact that

$$\tanh\sqrt{(iy)} = \tanh(u+iu) = \frac{\sinh 2u + i\sin 2u}{\cosh 2u + \cos 2u} \text{ (See Pennisi [11])}$$

we obtain

$$\frac{1}{iy}\exp[iy\xi - t\sqrt{(iy)}\tanh\sqrt{(iy)}]$$

$$= \frac{1}{y}\exp[-t\,B_1(u)].\{\sin[y\xi - t\,B_2(u)] - i\cos[y\xi - t\,B_2(u)]\} \qquad (A.13)$$

$$B_1(u) = \frac{u(\sinh 2u - \sin 2u)}{\cosh 2u + \cos 2u} \qquad (A.14)$$

$$B_2(u) = \frac{u(\sinh 2u + \sin 2u)}{\cosh 2u + \cos 2u}. \qquad (A.15)$$

We can also obtain $\varphi(iy,\eta)$ in the form

$$\varphi(iy,\eta) = \varphi_R(u,\eta) + i\varphi_I(u,\eta) \qquad (A.16)$$

where

$$\varphi_R(u,\eta) = \frac{\cosh\eta u\cos(2-\eta)u + \cos\eta u\cosh(2-\eta)u}{\cosh 2u + \cos 2u} \qquad (A.17a)$$

and

$$\varphi_I(u,\eta) = -\frac{\sinh\eta u\sin(2-\eta)u + \sinh(2-\eta)u\sin\eta u}{\cosh 2u + \cos 2u}. \qquad (A.17b)$$

From (A.12), (A.13), (A16), and (A.18), we obtain

$$\mathrm{Re}[e^{iy\xi} f(iy)] = \frac{1}{y}\exp[-t\,B_1(u)]\{\sin[y\xi - t\,B_2(u)].\varphi_R(u,\eta) + \cos[y\xi - tB_2(u)] \times \varphi_I(u,\eta)\}. \qquad (A.19)$$

Substituting into equation (A.11) and changing the variable of integration from y to u with the substitution $u = \sqrt{(y/2)}$ gives

$$U(\xi,t,\eta) = \frac{1}{2} + \frac{2}{\pi}\int_0^{\infty}\frac{1}{u}\{\exp[-tB_1(u)]\}\{\sin[2\xi u^2 - tB_2(u)].\varphi_R(u,\eta) + \cos[2\xi u^2 - tB_2(u)].\varphi_I(u,\eta)\}\,du. \qquad (A.20)$$

As $u \to 0$ the above integrand goes to zero, so that the singularity in the integrand at $u = 0$ is removable. As $u \to \infty$, $B_1(u) \to u$. The functions $\varphi_R(u,\eta)$ and $\varphi_I(u,\eta)$ are continuous and bounded for $0 < u < \infty$. Hence the integral converges because of the exponential term.

For $\eta = 0$ we obtain

$$U(\xi, t, 0) = \frac{1}{2} + \frac{2}{\pi} \int_0^\infty \frac{1}{u} \{\exp[-tB_1(u)]\} \sin[2\xi u^2 - tB_2(u)] \, du \quad (A.21)$$

which corresponds to the result given by Rosen except for the differences caused by the spherical geometry he considered. It can easily be shown that

$$U(\xi, 0, 0) = \frac{1}{2} + \frac{2}{\pi} \int_0^\infty \frac{\sin v}{v} dv = 1;$$

thus, the inlet condition is satisfied. It is not readily apparent, however, that equation (A.20) reduces to zero when $\xi = 0$. This has been checked numerically. Computer results were less than 10^{-5} for $\xi = 0$.

Résumé—La dynamique des échangeurs à plaques parallèles par récupération est étudiée à l'aide d'un modèle dans lequel la résistance au transport de chaleur est due à la résistance à la diffusion dans le fluide dans la direction transversale à l'écoulement. Le nombre de Nusselt résultant est une fonction du temps aussi bien que de la distance longitudinale mise sans dimensions à partir de l'entrée de l'échangeur et n'est pas constant, comme il est supposé dans le problème traditionnel du récupérateur. Cependant, pour des distances de l'entrée raisonnablement grandes, les profils de température sont en accord assez raisonnable avec ceux obtenus à partir de la solution traditionnelle de Anzelium–Schumann en dépit du fait que les nombres de Nusselt locaux deviennent très grands pour des temps assez longs.

Zusammenfassung—Das Übertragungsverhalten eines Paralleplatten-Regenerators wird anhand eines Modells untersucht, das den Wärmewiderstand auf einen reinen Leitwiderstand senkrecht zur Strömungsrichtung zurück führt.

Die resultierende Nusselt-Zahl ist sowohl eine Funktion der Zeit, als auch der Entfernung vom Regeneratoreintritt, demnach also keine Konstante, wie beim traditionellen Regeneratorproblem angenommen wird.

Für nicht zu kleine dimensionslose Entfernungen vom Eintritt stimmen die errechneten Temperaturprofile allerdings recht gut mit jenen nach der klassischen Anzelium–Schumann–Lösung überein, trotz der Tatsache, dass die lokalen Nusselt-Zahlen für grosse Zeiten sehr gross werden.

Аннотация—Изучается динамика параллельного пластинчатого регенеративного теплообменика путем использования модели, в которой теплообмен определяется диффузией в жидкости, текущей поперечно основному потоку. Результирующее число Нуссельта зависит от времени, а также от продольного расстояния от входа в теплообменник и не является постоянным, как это обычно предполагается в традиционной задаче о регенераторах. Однако, для весьма больших безразмерных продольных расстояний от входа температурные профили хорошо согласуются с уже ранее полученными профилями из известного решения Анзелиума–Шумаха несмотря на тот факт, что локальные числа Нуссельта становятся очень большими для больших времен.